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Solvable cases in the problem of motion of a heavy rotationally symmetric ellipsoid on a perfectly rough plane

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Abstract

We consider a classical problem of nonholonomic system dynamics – the problem of motion of a rotationally symmetric body on a fixed perfectly rough plane in a case when the moving body is a rotationally symmetric ellipsoid. Using the Kovacic algorithm we found several conditions under which equations of motion of the ellipsoid can be completely solved in quadratures.

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Keywords: Rotationally symmetric ellipsoid; Nonholonomic system; Kovacic algorithm; Liouvillian solutions.

1. Introduction

The problem of rolling without sliding of a heavy rotationally symmetric body on a fixed horizontal plane is a classical problem of nonholonomic mechanics. In 1897, S.A. Chaplygin in his paper[1] proved that the solution of this problem is reduced to the integration of the second-order linear differential equation with respect to the component of the angular velocity of the body in the projection on its axis of symmetry. However, a solution of this differential equation cannot always be found. In a case when the moving body is a nonhomogeneous dynamically symmetric ball, the general solution of the corresponding equation is expressed in terms of elementary functions[1]. In a case of motion of a circular disk or a hoop on a horizontal plane, the general solution of the mentioned equation is expressed in terms of a hypergeometric series[1]. In the paper[2], Kh.M. Mushtari continued the investigation of the problem of motion of a heavy rotationally symmetric body on a perfectly rough horizontal plane. Under additional condition imposing restrictions on a shape of the rolling body and a mass distribution in it, two new particular cases have been found, when the motion of the body can be investigated completely. In the first case the moving body is bounded by the surface formed by rotating a parabolic arc about an axis passing through its focus, and in the second case the moving body is a rotationally symmetric paraboloid. For other rotationally symmetric bodies moving without sliding on a horizontal plane, an exact solution of the corresponding second-order linear differential equation is unknown.

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In this paper we try to find the exact solution of the corresponding second-order linear differential equation in a case when the moving body is a rotationally symmetric ellipsoid. To find this solution we use the so-called Kovacic algorithm.

In 1986, American mathematician J. Kovacic proposed the algorithm[3] for finding a general solution of a second-order linear differential equation with variable coefficients for a case when this solution can be expressed in terms of so-called liouvillian functions[3,4]. Recall that liouvillian functions are functions that are built up from the rational functions by algebraic operations, taking exponentials and by integration. If a linear differential equation has no liouvillian solutions, the Kovacic algorithm also allows to ascertain that fact.

Using the Kovacic algorithm in the problem of motion of a rotationally symmetric ellipsoid on a perfectly rough horizontal plane we found several cases when the corresponding second-order linear differential equation has liouvillian solutions under additional restrictions on the parameters of the system. Physical admissibility of these additional restrictions is discussed.

The paper is organized as follows. In Section 2 we give the detailed problem formulation following the papers by Chaplygin[1] and Mushtari[2]. We derive also the linear second-order differential equation for a general rotationally symmetric body. In Section 3 we discuss specific features of application of the Kovacic algorithm to second-order linear differential equations. Finally in Section 4 we present our own results obtained in the problem of motion of a heavy rotationally symmetric ellipsoid on a perfectly rough horizontal plane.

2. General Problem Formulation

Let us consider the general problem of motion of a rotationally symmetric rigid body on a fixed perfectly rough horizontal plane. Suppose that the centre of mass G of the body is situated on the symmetry axis $G\zeta$, and moments of inertia about principal axes of inertia $G\xi$ and $G\eta$ perpendicular to $G\zeta$ are equal to each other. The body moves in presence of the homogeneous gravity field.

Let $Oxyz$ be the fixed coordinate frame with the origin in the supporting plane Oxy. Let θ be the angle between the symmetry axis of the body and the vertical. The distance GQ of the centre of mass over the plane Oxy is a function of angle θ[1,2]:

$$GQ = f(\theta). \tag{1}$$

Let β be the angle between the meridian $M\zeta$ of the body and a certain fixed meridian plane, and α is the angle between horizontal tangent MQ of the meridian $M\zeta$ and the Ox-axis. The position of the body is completely determined by the angles α, β and θ and by the x and y coordinates of the point M.

Let us specify now the position of the coordinate system $G\xi\eta\zeta$. Suppose that the $G\xi$-axis is always situated in the plane of the vertical meridian $M\zeta$ while the $G\eta$-axis is perpendicular to this plane (Fig. 1). In this case the coordinate system $G\xi\eta\zeta$ moves both in the space and in the body. Denote by ξ, η, ζ the coordinates of the point of contact M of the body with the supporting plane in the coordinate system $G\xi\eta\zeta$. Then $\eta = 0$ and[1,2]:

$$\xi = -f(\theta)\sin\theta - f'(\theta)\cos\theta, \quad \zeta = -f(\theta)\cos\theta + f'(\theta)\sin\theta, \tag{2}$$

where $()'$ is a derivative of function $f(\theta)$ with respect to θ. Thus we can completely characterize the surface of the moving body using the function $f(\theta)$.

Let the velocity $\mathbf{v}$ of the centre of mass G, the angular velocity vector $\boldsymbol{\omega}$ of the body, the angular velocity vector $\boldsymbol{\Omega}$ of the coordinate system $G\xi\eta\zeta$, and the reaction of the plane $\mathbf{R}$ are specified in the system $G\xi\eta\zeta$ by the components $v_\xi, v_\eta, v_\zeta;\ p, q, r;\ \Omega_\xi, \Omega_\eta, \Omega_\zeta$ and R_ξ, R_η, R_ζ, respectively. Let m be the mass of the body, A_1 – its moment of inertia about axes $G\xi$ and $G\eta$, and A_3 – its moment of inertia about the symmetry axis.

Since the $G\zeta$ axis is fixed in the body, then

$$\Omega_\xi = p, \quad \Omega_\eta = q. \tag{3}$$

The third component Ω_ζ can be easily expressed through p; indeed, the plane $G\xi\zeta$ is always vertical, i.e. the projection of the angular velocity of the axes $G\xi\eta\zeta$ on MQ equals to zero, therefore

$$\Omega_\zeta = \Omega_\xi \cot\theta = p\cot\theta. \tag{4}$$

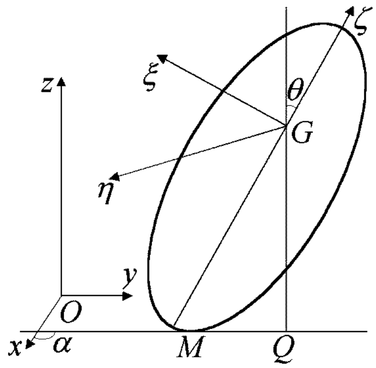

Fig. 1. Motion of a rotationally symmetric body: basic coordinate systems.

The velocity of the point of contact M is zero therefore

$$\mathbf{v} + \left[\omega \times \overrightarrow{GM}\right] = \mathbf{0}. \tag{5}$$

Theorems of change of momentum and angular momentum of the body give:

$$m\dot{\mathbf{v}} + m\left[\boldsymbol{\Omega} \times \mathbf{v}\right] = -m\mathbf{g} + \mathbf{R}, \tag{6}$$

$$\dot{\mathbf{K}} + \left[\boldsymbol{\Omega} \times \mathbf{K}\right] = \left[\overrightarrow{GM} \times \mathbf{R}\right]. \tag{7}$$

Here $\mathbf{g}$ is the acceleration due to gravity and $\mathbf{K}$ is the angular momentum of the body with respect to its center of mass. We can write equations (5)-(7) in scalar form:

$$v_\xi + q\zeta = 0, \quad v_\eta + r\xi - p\zeta = 0, \quad v_\zeta - q\xi = 0; \tag{8}$$

$$\begin{gathered} \frac{dv_\xi}{dt} + \Omega_\eta v_\zeta - \Omega_\zeta v_\eta = -\mathrm{g}\sin\theta + \frac{R_\xi}{m}, \quad \frac{dv_\eta}{dt} + \Omega_\zeta v_\xi - \Omega_\xi v_\zeta = \frac{R_\eta}{m}, \\ \frac{dv_\zeta}{dt} + \Omega_\xi v_\eta - \Omega_\eta v_\xi = -\mathrm{g}\cos\theta + \frac{R_\zeta}{m}; \end{gathered} \tag{9}$$

$$\begin{gathered} A_1\frac{dp}{dt} + A_3 r\Omega_\eta - A_1 q\Omega_\zeta = -\zeta R_\eta, \quad A_1\frac{dq}{dt} + A_1 p\Omega_\zeta - A_3 r\Omega_\xi = \zeta R_\xi - \xi R_\zeta, \\ A_3\frac{dr}{dt} + A_1 q\Omega_\xi - A_1 p\Omega_\eta = \xi R_\eta. \end{gathered} \tag{10}$$

Excluding R_ξ, R_η and R_ζ from the equations (9), (10) and using some simplifications based on (2)-(4) and (8) we obtain three equations:

$$\begin{gathered} \left[A_1 + m\left(\xi^2 + \zeta^2\right)\right]\frac{dq}{dt} = m\mathrm{g}f'(\theta) + (A_3 r - A_1 p\cot\theta)\,p - mp\,(\zeta\cot\theta + \xi)(p\zeta - r\xi) - mq\left(\xi\frac{d\xi}{dt} + \zeta\frac{d\zeta}{dt}\right), \\ A_1\frac{dp}{dt} + A_3\frac{\zeta}{\xi}\frac{dr}{dt} = (A_1 p\cot\theta - A_3 r)\,q, \\ \frac{d}{dt}(p\zeta - r\xi) - \frac{A_3}{m\xi}\frac{dr}{dt} = (\zeta\cot\theta + \xi)\,pq. \end{gathered} \tag{11}$$

Here ξ and ζ are functions of θ, determined by (2). Having added to (11) the obvious equation

$$q = -\frac{d\theta}{dt}, \tag{12}$$

we obtain the closed system of four differential equations with four unknown functions of time p, q, r and θ.

Equations (11)-(12) possess the energy integral:

$$E = \frac{A_1}{2} p^2 + \frac{1}{2}\left(A_1 + m\left(\xi^2 + \zeta^2\right)\right) q^2 + \frac{A_3}{2} r^2 + \frac{m}{2}(p\zeta - r\xi)^2 + mgf(\theta) = c_0 = \text{const.}$$

Suppose $\theta \neq$ const. Then using (12) we can change the independent variable t in the second and in the third equation of the system (11) to the new independent variable θ. As a result we obtain:

$$\begin{aligned} A_1 \frac{dp}{d\theta} + A_3 \frac{\zeta}{\xi}\frac{dr}{d\theta} &= -A_1 p \cot\theta + A_3 r, \\ \zeta \frac{dp}{d\theta} - \frac{A_3 + m\xi^2}{m\xi}\frac{dr}{d\theta} &= -(\zeta\cot\theta + \xi + \zeta')p + \xi' r. \end{aligned} \tag{13}$$

From the system (13) we can obtain for r the second-order linear differential equation

$$\frac{d^2 r}{d\theta^2} + \left[\frac{\cos\theta}{\sin\theta} + \frac{3m\left(A_1\xi\xi' + A_3\zeta\zeta'\right)}{\Delta} - \frac{\frac{d}{d\theta}\left(\xi\left(\xi + \zeta'\right)\right)}{\xi\left(\xi + \zeta'\right)}\right]\frac{dr}{d\theta} + \frac{m\xi\left(\xi + \zeta'\right)}{\Delta\sin\theta}\left[\frac{d}{d\theta}\left(\frac{\left(A_1\xi' - A_3\zeta\right)\sin\theta}{\xi + \zeta'}\right) - A_3\sin\theta\right] r = 0, \tag{14}$$

$\Delta = A_1 A_3 + A_1 m\xi^2 + A_3 m\zeta^2$.

Integration of the system (13) or the equation (14) gives the expressions for p and r as functions of θ with two arbitrary constants. Further integration of the problem finishes in quadratures. However the exact solution of the equation (14) is known only in the case when the moving body is a nonhomogeneous dynamically symmetric ball or a circular disk [1]. The present paper deals with the study of motion of a rotationally symmetric ellipsoid on a perfectly rough horizontal plane.

3. The specific features of application of the Kovacic algorithm to differential equations

In this section we discuss specific features of application of the Kovacic algorithm to second order linear differential equations. The algorithm under consideration provides obtention of general solution of second order linear differential equation in explicit form, expressed by liouvillian functions [3], or show that the equation has no such solution. The step-by-step execution of the algorithm is concerned with cumbersome though uncomplicated calculations. Therefore we are not going to deepen into details of the algorithm: they are particularized in the original work of J. Kovacic [3]. We will provide with the initial concepts.

The second order linear differential equation which can be investigated by the Kovacic algorithm is supposed to have the following form:

$$\frac{d^2 z}{dx^2} + a(x)\frac{dz}{dx} + b(x) z = 0, \tag{15}$$

where $a(x)$ and $b(x)$ are rational functions of variable x. Through the variable change

$$y = \exp\left(\frac{1}{2}\int a(x)\,dx\right) z$$

the equation (15) can be rewritten as follows:

$$\frac{d^2 y}{dx^2} = R(x) y, \quad R(x) = \frac{1}{2}\frac{da(x)}{dx} + \frac{1}{4}(a(x))^2 - b(x). \tag{16}$$

Then the rational function $R(x)$ is expanded in partial fractions and its finite poles are analyzed as well as the its pole at $x = \infty$. On the basis of the obtained data we conclude about the existence of liouvillian solutions of the equation (16).

In the next section we consider the problem of motion of a rotationally symmetric ellipsoid rolling on a fixed perfectly rough plane. We will obtain the corresponding equation (14) and transform it into the form (16). After that we will briefly inform about the results of application of the Kovacic algorithm for finding liouvillian solutions.

4. Motion of a rotationally symmetric ellipsoid

Suppose that the moving rigid body is a rotationally symmetric ellipsoid with the semi-axes a_1 and a_3. From (1) and (2) we obtain:

$$f = f(\theta) = \sqrt{a_1^2 \sin^2\theta + a_3^2 \cos^2\theta},$$

$$\xi = -\frac{a_1^2 \sin\theta}{\sqrt{a_1^2 \sin^2\theta + a_3^2 \cos^2\theta}}, \quad \zeta = -\frac{a_3^2 \cos\theta}{\sqrt{a_1^2 \sin^2\theta + a_3^2 \cos^2\theta}}.$$

In this case equation (14) takes a form:

$$\frac{d^2 r}{d\theta^2} + b_1(\theta)\frac{dr}{d\theta} + b_2(\theta) r = 0, \tag{17}$$

$$b_1(\theta) = \frac{\cos\theta}{\sin\theta} - \frac{4a_3^2\cos\theta}{\left(a_1^2\sin^2\theta + a_3^2\cos^2\theta\right)\sin\theta} + \frac{3\left(A_1 a_1^2 - A_3 a_3^2\right) m a_1^2 a_3^2 \sin\theta\cos\theta}{\left(a_1^2\sin^2\theta + a_3^2\cos^2\theta\right)\left(\left(A_1 + m a_3^2\right) A_3 a_3^2 \cos^2\theta + \left(A_3 + m a_1^2\right) A_1 a_1^2 \sin^2\theta\right)},$$

$$b_2(\theta) = -\frac{m a_1^2\left(\left(a_3^2 - a_1^2\right)^2 A_3 \sin^4\theta + a_3^2\left(A_1 a_1^2 - A_3 a_3^2\right)\left(1 + \cos^2\theta\right)\right)}{\left(a_1^2\sin^2\theta + a_3^2\cos^2\theta\right)\left(\left(A_1 + m a_3^2\right) A_3 a_3^2 \cos^2\theta + \left(A_3 + m a_1^2\right) A_1 a_1^2 \sin^2\theta\right)}.$$

In the equation (17) we change the independent variable by the formula $\cos^2\theta = x$ and introduce the following dimensionless parameters:

$$\frac{A_3}{A_1} = A \in (0,\, 2), \quad \frac{m a_1^2}{A_1} = B, \quad \frac{a_3^2}{a_1^2} = C, \quad \frac{1}{1 - C} = x_1, \quad \frac{A + B}{A + B - (1 + BC) AC} = x_2.$$

After this transformation equation (17) will take a form:

$$\frac{d^2 r}{dx^2} + b_1(x)\frac{dr}{dx} + b_2(x) r = 0, \tag{18}$$

$$b_1(x) = \frac{3x - 1}{2x(x - 1)} - \frac{2(x_1 - 1)}{(x - 1)(x - x_1)} + \frac{3(x_2 - x_1)}{2(x - x_1)(x - x_2)},$$

$$b_2(x) = \frac{(x_2 - x_1)\left(Ax^2 - \left((A - 1)\left(x_1^2 - 2x_1 + 3\right) - x_1 + 3\right)x - ((A - 1)(x_1 - 2) - 1)x_1\right)}{4(x_1 - 1)((A - 1)x_1 - A)(x - 1)(x - x_1)(x - x_2)x}.$$

Now let us set

$$y(x) = r(x)\exp\left(\frac{1}{2}\int b_1(x)\,dx\right).$$

Then equation (18) becomes

$$\frac{d^2 y}{dx^2} = \left(\frac{1}{2}\frac{db_1(x)}{dx} + \frac{b_1^2(x)}{4} - b_2(x)\right) y = E(x)\, y, \tag{19}$$

$$E(x) = \frac{\beta_0}{x} + \frac{\alpha_0}{x^2} + \frac{\beta_1}{x - 1} + \frac{\alpha_1}{(x - 1)^2} + \frac{\beta_2}{x - x_1} + \frac{\alpha_2}{(x - x_1)^2} + \frac{\beta_3}{x - x_2} + \frac{\alpha_3}{(x - x_2)^2},$$

$$\alpha_1 = \frac{3}{4}, \quad \alpha_0 = \alpha_2 = \alpha_3 = -\frac{3}{16},$$

$$\beta_0 = \frac{(2x_1x_2 - x_1 - 3x_2)(A-1)}{8x_2((A-1)x_1 + A)} + \frac{A\left((x_2-1)(x_1+1) - (x_1-1)^2(2x_2-1)\right)}{8x_1x_2(x_1-1)((A-1)x_1 - A)},$$

$$\beta_1 = -\frac{x_1x_2 - 2x_1 - 4x_2 + 5}{4(x_1-1)(x_2-1)},$$

$$\beta_2 = \frac{(3x_1x_2 - 4x_1 + x_2)(x_1-1)A - 3x_1(x_1x_2 - 2x_1 + x_2)}{8x_1(x_1-1)(x_1-x_2)((A-1)x_1 - A)},$$

$$\beta_3 = \frac{A(x_2-1)^2}{4x_2(x_1-1)(x_1-x_2)((A-1)x_1-A)} + \frac{(x_1x_2 + x_1 - 2)A - (A-1)\left(2x_2^2 + x_1x_2 + x_1 - 4x_2\right)x_1}{8x_2(x_2-1)(x_1-x_2)((A-1)x_1-A)}.$$

The Laurent series expansion of $E(x)$ at $x = \infty$ is given by

$$E(x)|_{x=\infty} \approx \frac{b_\infty}{x^2} + O\left(\frac{1}{x^3}\right),$$

$$b_\infty = -\frac{3}{16} + \frac{A(x_1-x_2)}{(x_1-1)((A-1)x_1-A)}.$$

Function $E(x)$ is a rational function. It has the four poles located at $x = 0$, $x = 1$, $x = x_1$ and $x = x_2$. Every of these poles has order 2. The order of $E(x)$ at $x = \infty$ is at least 2. Therefore we can try to find the liouvillian solutions of the equation (19) using the Kovacic algorithm[3]. Direct application of the Kovacic algorithm to the equation (19) gives the following results about existence of liouvillian solutions of (19).

Theorem 1. *If*

$$\sqrt{1+4b_\infty} \notin \mathbb{Q}$$

then equation (19) *don't possess liouvillian solutions.*

Theorem 2. *If* $b_\infty = 0$ *then equation* (19) *don't possess liouvillian solutions.*

Theorem 3. *If the rotationally symmetric ellipsoid rolling on a perfectly rough plane is homoneneous*

$$A_1 = \frac{m\left(a_1^2 + a_3^2\right)}{5}, \quad A_3 = \frac{2ma_1^2}{5}$$

then equation (19) *don't possess liouvillian solutions.*

However there are also the cases, when equation (19) possess liouvillian solutions. Let us consider here only the two most simple cases.

Theorem 4. *Equation* (19) *possesses liouvillian solution when the parameters of the problem satisfy the following condition*

$$b_\infty = \frac{5}{16} \quad or \quad A = \frac{2x_1(x_1-1)}{2x_1^2 + 2 - 5x_1 + x_2}. \tag{20}$$

Condition (20) can be rewritten in the form:

$$\frac{A_3}{A_1} = \frac{2ma_1^4}{2A_1a_3^2 - 2A_1a_1^2 + ma_1^2a_3^2 - ma_1^4 + 2ma_3^4}. \tag{21}$$

Expression in the right-hand side of equation (21) should belong to the interval (0, 2). Therefore the following condition on the parameters of the system should take place:

$$2A_1a_3^2 - 2A_1a_1^2 + ma_1^2a_3^2 - 2ma_1^4 + 2ma_3^4 > 0.$$

This condition is always satisfied in a case of the oblong ellipsoid $\left(a_3^2 > a_1^2\right)$. In a case of the oblate ellipsoid $\left(a_1^2 > a_3^2\right)$ this condition is satisfied only if the parameter C belongs to the interval

$$C = \frac{a_3^2}{a_1^2} \in \left(\frac{\sqrt{17}-1}{4},\ 1\right).$$

When the parameters of the problem satisfy the condition (20), the liouvillian solution of the equation (18) takes the form:

$$r(x) = \sqrt{\frac{P_2(x)}{x - x_2}}\left(C_1 e^{\int \omega(x)dx} + C_2 e^{-\int \omega(x)dx}\right),$$

$$\omega(x) = \frac{(x-1)}{2P_2(x)}\sqrt{\frac{(x_1x_2 - 2x_1 + 1)(4x_1 - x_2 - 2)}{x(x - x_1)(x - x_2)}},$$

$$P_2(x) = x^2 - x_2 x + x_1 x_2 - 2x_1 + 1.$$

Thus in the case when the parameters of the system satisfy the condition (20) the problem of motion of a rotationally symmetric ellipsoid on a perfectly rough plane can be completely solved in quadratures.

Another case, when equation (19) possesses liouvillian solutions is the following.

Theorem 5. *Equation* (19) *possesses liouvillian solution when the parameters of the problem satisfy the following condition*

$$b_\infty = \frac{21}{16} \quad or \quad A = \frac{6x_1(x_1 - 1)}{6x_1^2 - 13x_1 + x_2 + 6}. \tag{22}$$

Condition (22) can be rewritten in the form:

$$\frac{A_3}{A_1} = \frac{6ma_1^4}{6A_1a_3^2 - 6A_1a_1^2 + 6ma_3^4 + ma_1^2a_3^2 - ma_1^4}. \tag{23}$$

Expression in the right-hand side of equation (23) should belong to the interval (0, 2). Therefore the following condition on the parameters of the system should take place:

$$6A_1a_3^2 - 6A_1a_1^2 + 6ma_3^4 + ma_1^2a_3^2 - 4ma_1^4 > 0.$$

This condition is always satisfied in a case of the oblong ellipsoid $\left(a_3^2 > a_1^2\right)$. In a case of the oblate ellipsoid $\left(a_1^2 > a_3^2\right)$ this condition is satisfied only if the parameter C belongs to the interval

$$C = \frac{a_3^2}{a_1^2} \in \left(\frac{\sqrt{97}-1}{12},\ 1\right).$$

When the parameters of the problem satisfy the condition (22), the liouvillian solution of the equation (18) takes the form:

$$r(x) = \sqrt{\frac{P_3(x)}{x - x_2}}\left(C_1 e^{\int \omega(x)dx} + C_2 e^{-\int \omega(x)dx}\right),$$

$$\omega(x) = \frac{(x-1)}{P_3(x)}\sqrt{\frac{3\left(3x_1^2 + 10x_1x_2 - x_2^2 - 20x_1 - 4x_2 + 12\right)K}{x(x - x_1)(x - x_2)}},$$

$$P_3(x) = 9x^3 - 3(3x_1 + 4x_2 - 2)x^2 + \left(4x_2^2 + 3x_1x_2 + 12x_1 - 4x_2 - 8\right)x - K,$$

$$K = (x_2 - 2)\left(3x_1^2 + 4x_1x_2 - 20x_1 + 12\right).$$

This is the second case when the problem of motion of a rotationally symmetric ellipsoid on a perfectly rough plane can be completely solved in quadratures.

Existence of the exact solution of the equation (18) allows to give an exhaustive analysis of motion of the ellipsoid on a perfectly rough plane. We shall try to make such analysis in the future.

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